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A Note on Multiple-Entry Finite Automata

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Recently, Gill and Kou [1] introduced the multiple-entry finite automaton (mefa). It is a deterministic finite automaton (dfa) with no specified initial state (i.e., it accepts if there is a sequence of transitions, starting from *any* state and ending in a final state). They showed that the class of languages accepted by mefa's is the class of regular languages closed under suffix, gave an algorithm to test for the property, and tried to show that mefa descriptions may be very economical by exhibiting a mefa with n states such that every equivalent dfa has at least 2^n states. This result, however, uses an alphabet of size 2^n, so if we measure size by number of states times alphabet size, only a small reduction is achieved. They asked whether this result holds for fixed alphabet size. For the sake of completeness, we settle their question. (It has been brought to our attention that another answer was found independently by Mr. P. A. S. Veloso.)

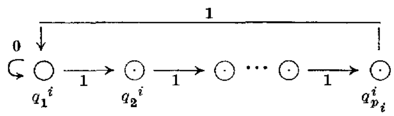
$M_i =$ 

FIG. 1. $\odot$ = accepting state.

Let $l > 0$, let p_i be the ith prime and let $N = \prod_{i=1}^{l} p_i$. Consider the n-state mefa M which consists of M_i, $1 \leqslant i \leqslant l$ of Fig. 1. $n = \sum_{i=1}^{l} p_i \leqslant cl^2 \cdot \log l$, by the prime number theorem. Let L be the language accepted by M and let $L' = L \cap 01^+$. If L can be accepted by a k-state dfa, then L' can be accepted by a $3k$-state dfa. But $L' = \{01^i \mid i \bmod N \neq 0\}$. By the Nerode theorem (cf. [2, Theorems 3.1, 3.2]), for $l \geqslant l_0$,

$$3k \geqslant N = \prod_{i=1}^{l} p_i \geqslant (p_{l/2})^{l/2} \geqslant (l/2)^{l/2} = 2^{l/2 \log(l/2)} \geqslant 4 \cdot 2^{\sqrt{n}}$$

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So if $l \geqslant l_0$, then every dfa equivalent to the n-state mefa M must have $2^{\sqrt{n}}$ states. We know of another n-state mefa such that every equivalent dfa must have 2^n states. However, the proof of this fact is slightly longer. Note also that every n-state mefa over a one-symbol alphabet can be easily transformed to an equivalent dfa with at most n states.

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References

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2. J. E. Hopcroft and J. D. Ullman, "Formal Languages and Their Relation to Automata," Addison–Wesley, Reading, Mass., 1969.